

Use refutation trees to show that each of these is valid. Then prove using the first 8 rules of the propositional calculus.

1. $P, (P \vee Q) \rightarrow S, (S \vee T) \rightarrow M \vdash M \vee (K \& T)$

2. $P \& S, P \leftrightarrow R, S \rightarrow T, T \rightarrow R \vdash R$

3. $P \rightarrow Q, Q \rightarrow P, (P \leftrightarrow Q) \rightarrow (R \rightarrow S), \sim\sim R \vdash S$

4. $(P \& M) \leftrightarrow (T \& R), \sim\sim T \& \sim\sim R \vdash (P \vee T) \vee (R \vee T)$

5*. $P \vee S, P \rightarrow (M \vee N), S \rightarrow (M \vee N) \vdash K \vee (M \vee N)$

6. $(P \rightarrow Q) \vee (S \rightarrow T), (P \rightarrow Q) \rightarrow (R \vee M), (S \rightarrow T) \rightarrow (R \vee M), R \rightarrow K, M \rightarrow K, K \rightarrow M \vdash K \leftrightarrow M$

*It may not have been clear from the book or my lecture that when you use $\vee I$, you may introduce the new formula on either side of the first formula.