Predicate calculus proofs with identity.

1. $Fa \vdash \forall x(x=a \rightarrow Fx)$

2. $\forall x \forall y(\sim Pxy \rightarrow \sim x=y) \vdash \exists x \exists y Pxy$

3. Translate and prove as a theorem:

   - Everything has something identical to it.

4. Translate and prove:

   - Bob is funny but Albert is not funny.
   - Therefore, not all things are identical to each other.

5. Translate and prove:

   - Whoever punked Ezekiel is a logician.
   - Only Mitch and Anise are logicians.
   - So, Mitch or Anise punked Ezekiel:

6. Translate and prove:

   - There is one and only one god. So if Aeger and Odin are both gods, then Aeger is Odin.

7. Translate and prove:

   - All who are pious worship God.
   - But there is only one who is pious.
   - So either God is not pious or God worships God.

8. Translate and prove:

   - There is at most one who is pious and that is Fred.
   - All those who are pious worship God.
   - But Fred worships nothing.
   - So, no one is pious.