

Predicate Calculus with all four rules for quantifiers.

1. $\exists x(Fx \& Gx), \forall y((Fy \& Gy) \rightarrow Kyy) \vdash \exists zKzz$
2. $\forall x\forall y(Fxy \rightarrow Fyx), \exists xFxa \vdash \exists xFax$
3. $\exists x(Fx \vee Gx) \vdash \exists xFx \vee \exists xGx$
4. $\sim\exists x(Fx \& Gx) \vdash \forall x(Fx \rightarrow \sim Gx)$
5. $\exists x\exists yFxy, \forall x\forall y(Fxy \rightarrow (Gx \vee Gy)) \vdash \exists x\exists y(Gx \vee Gy)$
6. $\exists xFx \vdash \sim\forall x\sim Fx$
7. $\forall x(Fx \rightarrow Gx), \exists x\sim Gx \vdash \sim\forall xFx$
8. $\sim\forall x\sim Fx \vdash \exists xFx$
9. $\forall x(Fx \rightarrow \forall y(Gy \rightarrow Hxy)), \exists x(Fx \& \exists y\sim Hxy) \vdash \exists x\sim Gx$
10. $\forall x(\exists yFxy \rightarrow \exists zFzx) \vdash \exists xFax \rightarrow \sim\forall z\sim Fza$
11. $\exists x(Fx \vee \forall yGxy) \vdash \exists yFy \vee \exists y(Gyy \& Gyb)$

Additional problems of various degrees of difficulty

12. $\forall x\forall y(Fxy \rightarrow Fyx) \vdash Fab \rightarrow Fba$
13. $\forall xFx \vee \forall yGy \vdash Fa \vee Gb$
14. $\forall x\forall y(Fxy \rightarrow Fyx), \exists xFxa \vdash \exists xFax$
15. $\forall xLxx \vdash \forall x\exists yLxy$
16. $\exists xGxx \rightarrow \forall x\forall yGxy, \sim Gac \vdash \sim Gbb$
17. $\sim\forall x(\exists yFy \rightarrow Gx) \vdash \sim\forall zGz$
18. $\exists x(\forall yGxy \vee \forall yGyx) \vdash \sim\forall x\forall y(\sim Gxy \& \sim Gyx)$
19. $\forall x((Fx \vee Hx) \rightarrow (Gx \& Kx)); \sim\forall x(Kx \& Gx) \vdash \exists x\sim Hx$
20. $\forall x\forall y(Gxy \leftrightarrow (Fy \rightarrow Hx)); \forall zGaz \vdash \exists xFx \rightarrow \exists xHx$
21. $\forall x(\exists yFxy \rightarrow \forall yFyx); \exists x\exists yFxy \vdash \forall x\forall yFxy$
22. $\forall x(Fxx \rightarrow Hx); \exists xHx \rightarrow \sim\exists yGy; \vdash \forall x(Gx \rightarrow \sim\exists zFzz)$