

Philosophy 60  
Test 7

Instructions: Use the propositional calculus to derive both of the following. You may use any rules or equivalences.

1. (10 points):  $\sim(P \vee M) \vee \sim(Q \rightarrow S) \vdash \sim(\sim S \& Q) \rightarrow (\sim P \& \sim M)$

1.  $\sim(P \vee M) \vee \sim(Q \rightarrow S)$  A
2.  $\sim(Q \rightarrow S) \vee \sim(P \vee M)$  1, COM
3.  $\sim(Q \rightarrow S) \vee (\sim P \& \sim M)$  2, DM
4.  $(Q \& \sim S) \vee (\sim P \& \sim M)$  3,  $\sim \rightarrow$
5.  $(\sim S \& Q) \vee (\sim P \& \sim M)$  4, COM
6.  $\sim\sim(\sim S \& Q) \vee (\sim P \& \sim M)$  5, DN
7.  $\sim(\sim S \& Q) \rightarrow (\sim P \& \sim M)$  6, MI

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(5 pts.)  $P \rightarrow (M \vee Q), \sim P \rightarrow (M \vee Q) \vdash \sim M \rightarrow Q$

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|-------------------------------------|----------------|
| 1. $P \rightarrow (M \vee Q)$       | A              |
| 2. $\sim P \rightarrow (M \vee Q)$  | A              |
| 3. $\mid \sim(P \vee \sim P)$       | H              |
| 4. $\mid \sim P \ \& \ \sim \sim P$ | 3, DM          |
| 5. $\sim \sim(P \vee \sim P)$       | 3-4 $\sim$ I   |
| 6. $P \vee \sim P$                  | 5, DN          |
| 7. $M \vee Q$                       | 6,2,1 $\vee$ E |
| 8. $\sim \sim M \vee Q$             | 7, DN          |
| 9. $\sim M \rightarrow Q$           | 8, MI          |