

Philosophy 60
Test 13

1. (10pts.)

$\vdash \forall x \forall y \forall z ((x=y \ \& \ y=z) \rightarrow x=z)$

- | | |
|--|----------------------|
| 1. $a = b \ \& \ b = c$ | H |
| 2. $a=b$ | 1, &E |
| 3. $b=c$ | 2, &E |
| 4. $a=c$ | 2,3 =E |
| 5. $(a=b \ \& \ b = c) \rightarrow a=c$ | 2-4, \rightarrow I |
| 6. $\forall z((a=b \ \& \ b = z) \rightarrow a = z)$ | 5, \forall I |
| 7. $\forall y \forall z((a=y \ \& \ y = z) \rightarrow a = z)$ | 6, \forall I |
| 8. $\forall x \forall y \forall z((x=y \ \& \ y = z) \rightarrow x = z)$ | 7, \forall I |

Philosophy 60

Test 13

2. (5 pts.) Translate and prove

Matilda is the only one dancing with Bob. So either someone is dancing with himself or Bob is not Matilda.

$Dmb \ \& \ \forall x(Dxb \rightarrow x=m) \ \vdash \ \exists xDxx \vee \sim b=m$

- | | | |
|-----|---|----------------|
| 1. | $Dmb \ \& \ \forall x(Dxb \rightarrow x=m)$ | A |
| 2. | Dmb | 2, &E |
| 3. | $\forall x(Dxb \rightarrow x=m)$ | 2, &E |
| 4. | $\sim(\exists xDxx \vee \sim b=m)$ | H(\sim I) |
| 5. | $\sim\exists xDxx \ \& \ \sim\sim b=m$ | 4, DM |
| 6. | $\sim\exists xDxx$ | 5, &E |
| 7. | $\sim\sim b=m$ | 5, &E |
| 8. | $\forall x\sim Dxx$ | 6, QE |
| 9. | $\sim Dbb$ | 8, \forall E |
| 10. | $b=m$ | 7, DN |
| 11. | Dbb | 2,10 =E |
| 12. | $Dbb \ \& \ \sim Dbb$ | 9,11 &I |
| 13. | $\sim\sim(\exists xDxx \vee \sim b=m)$ | 4-12, \sim I |
| 14. | $\exists xDxx \vee \sim b=m$ | 13, DN |

Philosophy 60
Test 13