

Philosophy 60  
Test 12 solution

Derive the following.

1. (10pts.)

$$\vdash \sim\forall x(Fx \& Gx) \rightarrow \sim\forall y\sim(Fy \rightarrow \sim Gy)$$

|    |                                                                                 |                      |
|----|---------------------------------------------------------------------------------|----------------------|
| 1. | $\sim\forall x(Fx \& Gx)$                                                       | H                    |
| 2. | $\exists x\sim(Fx \& Gx)$                                                       | 1, QE                |
| 3. | $\sim(Fa \& Ga)$                                                                | H( $\exists E$ )     |
| 4. | $\sim Fa \vee \sim Ga$                                                          | 3, DM                |
| 5. | $Fa \rightarrow \sim Ga$                                                        | 4, MI                |
| 6. | $\exists y(Fy \rightarrow \sim Gy)$                                             | 5, $\exists I$       |
| 7. | $\sim\forall y\sim(Fy \rightarrow \sim Gy)$                                     | 6, QE                |
| 8. | $\sim\forall y\sim(Fy \rightarrow \sim Gy)$                                     | 2, 3-7 $\exists E$   |
| 9. | $\sim\forall x(Fx \& Gx) \rightarrow \sim\forall y\sim(Fy \rightarrow \sim Gy)$ | 1-8, $\rightarrow I$ |

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2. (5 pts.)

$\sim\forall x\exists y(Fxy \leftrightarrow Fyx) \vdash \exists x\exists y(Fxy \& \sim Fyx)$

- |     |                                                              |                            |
|-----|--------------------------------------------------------------|----------------------------|
| 1.  | $\sim\forall x\exists y(Fxy \leftrightarrow Fyx)$            | A                          |
| 2.  | $\exists x\sim\exists y(Fxy \leftrightarrow Fyx)$            | 1, QE                      |
| 3.  | $\sim\exists y(Fay \leftrightarrow Fya)$                     | H $\exists E$              |
| 4.  | $\forall y\sim(Fay \leftrightarrow Fya)$                     | 2, QE                      |
| 5.  | $\sim(Fab \leftrightarrow Fba)$                              | 4, $\forall E$             |
| 6.  | $\sim\exists x\exists y(Fxy \& \sim Fyx)$                    | H ( $\sim I$ )             |
| 7.  | $\forall x\sim\exists y(Fxy \& \sim Fyx)$                    | 6, QE                      |
| 8.  | $\forall x\forall y\sim(Fxy \& \sim Fyx)$                    | 7, QE                      |
| 9.  | $\forall y\sim(Fay \& \sim Fya)$                             | 8, $\forall E$             |
| 10. | $\sim(Fab \& \sim Fba)$                                      | 9, $\forall E$             |
| 11. | $\sim Fab \vee \sim\sim Fba$                                 | 10, DM                     |
| 12. | $\sim Fab \vee Fba$                                          | 11, DM                     |
| 13. | $Fab \rightarrow Fba$                                        | 12, MI                     |
| 14. | $\forall y\sim(Fby \& \sim Fyb)$                             | 8, $\forall E$             |
| 15. | $\sim(Fba \& \sim Fab)$                                      | 14, $\forall E$            |
| 16. | $\sim Fba \vee \sim\sim Fab$                                 | 15, DM                     |
| 17. | $\sim Fba \vee Fab$                                          | 16, DM                     |
| 18. | $Fba \rightarrow Fab$                                        | 17, MI                     |
| 19. | $Fab \leftrightarrow Fba$                                    | 13, 18 $\leftrightarrow I$ |
| 20. | $(Fab \leftrightarrow Fba) \& \sim(Fab \leftrightarrow Fba)$ | 5, 19 $\& I$               |
| 21. | $\sim\sim\exists x\exists y(Fxy \& \sim Fyx)$                | 6-20 $\sim I$              |
| 22. | $\sim\sim\exists x\exists y(Fxy \& \sim Fyx)$                | 2, 3-21 $\exists E$        |
| 23. | $\exists x\exists y(Fxy \& \sim Fyx)$                        | 22 DN                      |

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