

Philosophy 60  
Test 10 solution

Derive the following using any rules of the propositional calculus and any of the following rules from the predicate calculus:  $\forall E$ ,  $\forall I$ ,  $\exists I$ , .

(Ψου ωιλλ νοτ νεεδ  $\exists E$  , βυτ ψου αρε φρεε το υσε ιτ.)

1. (10 pts.)

$\exists xFx \rightarrow \forall xFx$ ,  $Gb \rightarrow \forall zDzz \vdash (Fa \ \& \ \forall yGy) \rightarrow Daa$

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|----|---|----------------------|
| 1. | $\exists xFx \rightarrow \forall xFx$       | A                    |
| 2. | $Gb \rightarrow \forall zDzz$               | A                    |
| 3. | $Fa \ \& \ \forall yGy$                     | H                    |
| 4. | $\forall yGy$                               | 3, &E                |
| 5. | $Gb$  | 4, $\forall E$       |
| 6. | $\forall zDzz$                              | 2, 5 $\rightarrow E$ |
| 7. | $Daa$                                       | 6, $\forall E$       |
| 8. | $(Fa \ \& \ \forall yGy) \rightarrow (Daa)$ | 3-7, $\rightarrow I$ |

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2. (5 pts.)

$\forall x(\exists yFxy \rightarrow \forall z(\sim Gxz \vee \sim Gzx)), Fab, \forall zGaz \vdash \exists x\sim Gbx$

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|--|---------------------|
| 1. $\forall x(\exists yFxy \rightarrow \forall z(\sim Gxz \vee \sim Gzx))$ | A                   |
| 2. $Fab$   | A                   |
| 3. $\forall zGaz$  | A                   |
| 4. $\exists yFay \rightarrow \forall z(\sim Gaz \vee \sim Gza)$            | 1, $\forall E$      |
| 5. $\exists yFay$  | 2, $\exists I$      |
| 6. $\forall z(\sim Gaz \vee \sim Gza)$                                     | 4,5 $\rightarrow E$ |
| 7. $\sim Gab \vee \sim Gba$  | 6, $\forall E$      |
| 8. $Gab$   | 3, $\forall E$      |
| 9. $\sim\sim Gab$  | 8, DN               |
| 10. $\sim Gba$   | 7,9 DS              |
| 11. $\exists x\sim Gbx$  | 10, $\exists I$     |