Predicate Calculus

Rules for Identity

Identity

- So far we haven't done any proofs involving identity.
- This is because identity is a special relation that has its own inference rules.
- They are easy to learn, and adding them to the predicate calculus gives it a great deal more power.
- There are two rules for identity. They are:
  - Identity introduction (=I)
  - Identity elimination (=E)

Identity Introduction (=I)

- Identity introduction is based on the reflexivity of identity. A relation is reflexive if every object bears that relation to itself.

- Most relations that come to mind are not reflexive. Nothing is longer than itself. Nothing is smarter than itself. Nothing is inside itself. Some things love themselves, but not everything does. Some things lie to themselves, but not everything does.

- But identity is different. Everything is identical to itself.

- Hence, the rule of identity introduction simply says that on any line of a proof, for any name whatsoever, you may say that it is identical to itself.

Identity Introduction (=I) 2

- Formally the rule is:

- Identity Introduction (=I)

  For any name letter a, you may introduce $a = a$ on any line of a proof.

- In the justification you simply write '=I'. There are no line numbers to cite.

Example of =I

- Here is a simple example of the use of =I
- ├ ∀x x=x
- 1. a=a =I
- 2. ∀x x=x 1, ∀I
- You could also prove this indirectly by first hypothesizing ~∀x x=x, but the rule of =I allows you to prove the result directly.

Identity elimination (=E)

- The rule of identity elimination is based on the fact that any two things that are identical share all the same properties.
- Classically this is known as the "Indiscernibility of Identicals."
- If you know, for example, that Mark Twain is Samuel Clemens, and you also know that Mark Twain wrote *Tom Sawyer*, then you may infer that Samuel Clemens wrote *Tom Sawyer*.

Philosophical aside

- There is actually an interesting philosophical problem that arises with this principle.
- Suppose that you did not know that Mark Twain and Samuel Clemens are the same person.
- Then you could know that Mark Twain wrote *Tom Sawyer*, but not know that Samuel Clemens wrote *Tom Sawyer*.
- But that suggests that Mark Twain has a property that Samuel Clemens does not, namely, being known by you to have written *Tom Sawyer*.
- This is just one of many different paradoxes that arise with respect to intentional concepts like belief and knowledge.

Identity Elimination (=E) 2

- Formally the rule is:

- Identity Elimination (=E)

  If $\mathcal{P}$ is a wff containing a name letter a, then from either $a = b$ or $b = a$ we may substitute b for one or more occurrences of a in $\mathcal{P}$.

- In the justification you write '=E' and cite two lines:
  - the line on which the identity occurs;
  - the line containing the formula in which you are performing the substitution.

Example of =E

- Here is an example of the use of =E

- ∀x(Gx → ~Fx) ├ (Ga & Fb) → ~a=b

1.	∀x(Gx → ~Fx)	A
2.	(Ga & Fb)	H →I
3.	a=b	H ~I
4.	Ga → ~Fa	1, ∀E
5.	Ga	2, &E
6.	~Fa	4,5 →E
7.	~Fb	3,6 =E
8.	Fb	2, &E
9.	Fb & ~Fb	7,8 &I
10.	~a=b	3-9 ~I
11.	(Ga & Fb) → ~a=b	2-10 →I

Example of =I and =E

- And here is an example where both =E and =I are required.

- a=b & ~b=a ├ Fc

1. a=b & ~b=a A
2. a=b 1, &E
3. ~b=a 1, &E
4. ~a=a 2,3 =E
5. a=a =I
6. Fc 4,5 CONT

This might feel like a contradiction to you, but it's not.yet been shown to be. Remember that a contradiction is only one thing: the conjunction of a formula and it's negation.

Since line 2 tells us that a=b, we may substitute a for b in line 3.