Predicate Logic

Identity

Predication vs. Identity

- In English, the word "is" can be used in two importantly different ways. To see this, consider the following statements.

  - Tony Stark is clever.
  - Tony Stark is Iron Man.

- The first use is what we call the ‘is’ of predication. It simply predicates the property of cleverness to a person named Tony Stark: Ct

- The second use is what we call the ‘is’ of identity. It says that Tony Stark and Iron Man are the same guy. So "Tony Stark equals Iron Man" expresses a binary relation we might write as Eti

Identity

- But identity turns out to be such an important relation that in logic we give it special treatment and we will adopt specific rules for making inferences with propositions asserting this relation.
- We begin by adopting the familiar mathematical symbol = to express identity relations. And, unlike other relations, we will actually position the symbol in the familiar way between names, rather than in front of them. In other words, rather than =ab, we will write a=b.
- Here we are just going to focus on the new and interesting things we can say using the relation of identity. When we develop the predicate calculus we will adopt inference rules specific to it.

Identity

- First, it is important to understand that identity holds between individuals, not entire sentences, so we can say things like
  - $a=b$
  - $\exists x \forall y\ x=y$
- but not
  - $P=Q$
  - $Fa = Gb$

Cool stuff we can now say 1

- With identity we can say things that we didn't know how to say before. For example, believe it or not, prior to identity we couldn't say something like:
  - God exists.
- We might have tried to translate it as
  - Eg
- But this isn't correct, because existence is not a property, but a quantifier. Now, however, we can represent this sentence as
  - $\exists x\ x = g$

Cool stuff we can now say 2

- Moreover, we can now say things like:
  - There is only one god.
- Notice, here, that the word "god" is functioning as a predicate, not as a name. We translate this statement as:
  - $\exists x\ (Gx\ \&\ \forall y(Gy \rightarrow y = x))$
- It takes most people a little while to see how this proposition captures the statement that there is only one god. One way to understand it is by using your model theory. (This statement is true just in case Ga is true for some value of a and for any value of b in G$b \rightarrow b = a$, either Gb is false, or $b = a$.)
- More intuitively, it is telling us that there is some name, e.g., 'a' for which we can truly say Ga, but if we make this statement with respect to any other name, say Gb, then either that statement is false, or a and b are just different names for the same thing.

Cool stuff we can now say 3

- We do not need identity to say something like: There is at least one apple, since this is adequately expressed by $\exists x Ax$.
- However, to say that there are at least <u>two</u> apples, we need identity.
  - $\exists x \exists y\ ((Ax \& Ay) \& \sim x=y)$
- We also need identity to express the proposition that there is <u>at most</u> one apple.
  - $\forall x \forall y\ ((Ax \& Ay) \rightarrow x=y)$
- And this expresses the proposition that there are at most two apples:
  - $\forall x \forall y \forall z(((Ax \& Ay) \& Az)) \rightarrow (x=y \vee (y=z \vee x=z)))$

Cool stuff we can now say 4

- We can also now say things like
  - Tony is the only one who is having fun.
- This sentence doesn't just say that Tony is having fun. It says that Tony is having fun, <u>and</u> nobody else is. In other words, if anyone else is having fun, then that person is Tony.
  - $Ft \ \& \ \forall x \ (Fx \rightarrow x=t)$
- In English, this says: Tony is having fun, and for all x, if x is having fun, then x is identical to Tony.

Cool stuff we can now say 5

- Finally, the relation of identity also allows us to express superlatives like:
  - Alicia is the hottest.
- This is essentially to say that Alicia is hotter than everyone except Alicia herself.
  - $\forall x\,(\sim x = a \rightarrow Hax)$