Predicate Logic

The Semantics of Predicate Logic

Models

- The semantics of predicate logic makes use of models. A model specifies three things:
  - a universe of discourse
  - an interpretation of the names
  - an interpretation of the predicates
- A universe of discourse (or just 'universe' for short) is just the set of objects to which the names and predicates apply.
- You can have a universe of discourse that is just everything in our universe if you like. But you can also restrict your universe to certain kind of things. For example, we could say that we are only talking about people, or numbers, or cars, or a set of objects in a particular location like a room or a city.

Models 2

- Interpretations are just the English language meanings we permit for particular names and predicates. So, here is a fully specified model:

- Universe: Cities in California

- Names:
  - s = Sacramento
  - f = San Francisco
  - l = Los Angeles

- Predicates:
  - S = the relation that holds when x is south of y.
  - P= the relation that holds when x is more populated than y.
  - C= the class of objects that are capitol cities.

Models 3

- Having specified a model, we can state whether any particular sentence is true or false in the model.
- For example, is the following sentence true in the model just specified?
- Psl
- This sentence says: Sacramento is more populated than Los Angeles, so knowing this is false we can conclude that this proposition is false in the model.

Models 4

- More complicated propositions are quite a bit more tedious to evaluate. For example this proposition

- $\forall x(Cx \rightarrow \exists y Syx)$

- says that if anything is a capitol, then there is some city to the south of it. We know this is true, since Sacramento is the only capitol and 2 cities specified in the model are to the south of Sacramento. However, a complete evaluation will read as follows:

- The wff is true in the model if for every substitution of a city name for a in the proposition $Ca \rightarrow Sba$ results in a true formula for at least one substitution of a city name for b. There is only one capitol city, hence every substitution of a city name other than s for a will result in a false antecedent, making the entire conditional true. When s is substituted for a, and f is substituted for b, the proposition is also true, since San Francisco is to the south of Sacramento. Therefore, the proposition is true in the model.

Models 5

- Complete evaluations of complicated propositions in a model can become very tedious, even when it is quite easy to see what the truth value of the proposition is.

- In order to be able to fully evaluate all propositions, we need to add one more concept. To see why, consider a proposition like:

- ~∃xSxl

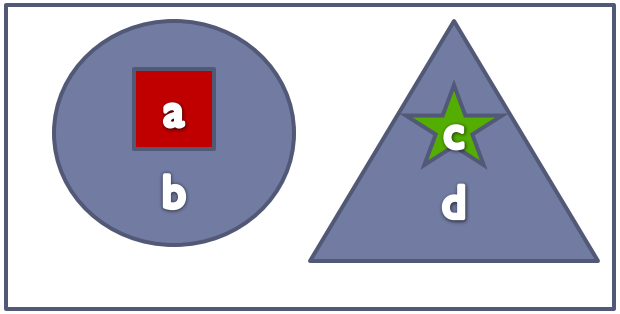
- This says that there is no city of California that is to the south of Los Angeles. We know this is false, but to show that it is false, we need to be able to talk about cities of California <u>not</u> specified in the model.

Models 6

- To do this, we introduce the idea of an a-variant.
- An a-variant in this case is just any city of California not explicitly named in the model. In general, an a-variant is an extension of the model that results from interpreting a new sentence letter.
- So a full evaluation of ~∃xSxl in the model is:
- This proposition is true in the model if Sal is false for every a and every a-variant. Sal is false for every a specified in the model, but it is not false for every a-variant. For example, if we specify the a-variant d= San Diego, the proposition Sdl is true in the model. Hence, the proposition ~∃xSxl is false.

Models 7

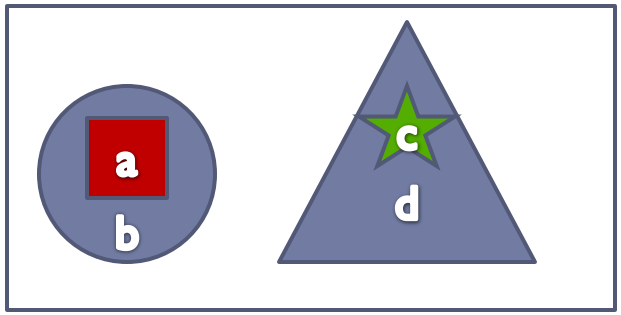
- It is important to understand how the evaluation of models works, but in this class we will not spend much time writing full evaluations of predicate logic sentences. However, we will design simple models to test our comprehension of predicate logic sentences. For example, we can draw a tiny universe like this:



- **Universe:** Two dimensional shapes inside the rectangle above.
- **Predicates:** R = red; B = blue; G= Green, T= triangular; C = circular; S = Square, R= Star, W = the relation 'within'.
- **Names:** a, b, c, d

- Then we can test our understanding of predicate logic by evaluating propositions.

Models 7



1. Sa & Rd
2. Cb v Cd
3. Wab → Wcb
4. ∃x(Rx & Wxa)
5. ∀y∃xWxy
6. ∃x∀yWyx → Ca